Effects of Permeability and Loading Rate on Dynamic Stiffness of Saturated Soil

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SYNOPSIS The dynamic stiffness of saturated soil influenced by drainage condition and loading rate is discussed. Saturated ground is dealt with as a two-phase material based on the formulation of Biot (Biot,1962). A thin layer element for describing the dynamic behavior of a fluid saturated porous layered medium is presented. The dynamic stiffness of the saturated two-phase grounds are given comparing with those for dry soil by using the developed formulation.

INTRODUCTION

Soil structures and foundations in Japanese coastal area have often suffered from great earthquakes. Earthquake engineering has been playing important role in coastal engineering works. In recent years it becomes more important to design the foundations of marine and offshore structures subjected to dynamic loading because of coastal development.

Most of the structures in coastal area are constructed on submerged soil deposits. Geotechnical engineering problems with those structures are often related to the behavior of pore water. Typical examples of these problems include consolidation of soft clay and liquefaction of loose sand. Particularly in liquefaction phenomena the relation between the permeability and loading rate is one of a critical factor to control the excess pore water pressure behavior including both accumulation and diffusion process. Two-phase nature of a material is, of course, very important. In the case of studying the dynamic stiffness of saturated ground in relation to the dynamic soil-structure interaction effects, it is also necessary to account for two-phase nature of the ground.

A great amount of work has been done to study dynamic stiffness for layered soil ground or half-space by using the thin layer element (e.g. Waas,1972). These results enable us to account for the soil properties varied with depth and for the radiation of waves into the far field by imposing boundary condition of the irregular region. However those analyses based on total stress analysis consider only the solid phase motion.

The object of this paper is to extend the foregoing thin layer element to saturated two-phase layered media and to study the dynamic stiffness of saturated sandy ground influenced by the permeability and loading rate.

Prepared by Kazama

BASIC EQUATIONS

In general Biot formulation is effective for describing the behavior of the fluid saturated porous media. The basic equations used here which are given by generalizing formulation of Biot were presented by Zienkiewicz & Shiomi(1984).

Full formulation of Biot (u-w form)

Equations of motion of mixture and fluid with static equilibirium removed are written by

$$[K]\begin{Bmatrix} u \\ w \end{Bmatrix} - [C]\begin{Bmatrix} \dot{u} \\ \dot{w} \end{Bmatrix} - [M]\begin{Bmatrix} \ddot{u} \\ \ddot{w} \end{Bmatrix} = 0 \quad (1)$$

$$[K]=\begin{bmatrix} L^T(D+\alpha^2 Q m m^T)L & \alpha Q \nabla\nabla^T \\ \alpha Q \nabla\nabla^T & Q \nabla\nabla^T \end{bmatrix} \quad (2)$$

$$[C]=\begin{bmatrix} 0 & 0 \\ 0 & k^{-1} \end{bmatrix} \quad (3)$$

$$[M]=\begin{bmatrix} \rho & \rho_f \\ \rho_f & \rho_f/n \end{bmatrix} \quad (4)$$

In these equations u is the displacement of solid phase, w is the average relative displacement of fluid phase to solid phase, ρ is the density of mixture, ρ_f is the density of fluid alone, $k=k0/\rho_f g$ is the coefficient of permeability per unit weight of fluid material, n is porosity of solid, and α,Q are constants of Biot formulation, satisfied the following equations.

$$\alpha = 1-(3\lambda+2G)/3K_s \quad (5)$$

$$1/Q = 1/K_f + (\alpha - n)/K_s \quad (6)$$

in which K_s is bulk modulus of solid, K_f is that of fluid, λ and G are Lame's elastic constants.

Total stress(σ) and pore pressure(π) are

$$\begin{Bmatrix}\sigma\\ \pi\end{Bmatrix}=\begin{bmatrix}(D+\alpha^2 Qmm^T)L & \alpha Qm\nabla^T\\ \alpha Q\nabla^T & Q\nabla^T\end{bmatrix}\begin{Bmatrix}u\\ w\end{Bmatrix} \quad (7)$$

in which $m^T=(1,1,0)$ in two dimensional case; L= matrix operator equivalent to the strain definition; and D= matrix of coefficients. And effective stress(σ') is

$$\sigma' = \sigma - m\pi \quad (8)$$

Undrained form

The relative displacement of fluid phase is negligible under undrained condition which indicates that the permeability is small enough comparing to loading rate. We can derive basic equation under undrained condition to eliminate the term related to variable w from equations (1) and (7). Therefore, equation of motion is

$$L(D+\alpha^2 Qmm^T)L\{u\}+\rho\{\ddot{u}\}=0 \quad (9)$$

and total stress and pore pressure are

$$\{\sigma\}=(D+\alpha^2 Qmm^T)L\{u\} \quad (10)$$

$$\pi=\alpha Q\nabla^T\{u\} \quad (11)$$

If Lame's elastic constant(λ_u) or Poisson's ratio (ν_u) under undrain condition is defined, i.e.

$$\lambda_u=\lambda+\alpha^2 Q = G\frac{2\nu_u}{(1-2\nu_u)} \quad (12)$$

we can write basic equations in the same way of just dry soil case. The only thing different from dry soil case is existence of the pore pressure. Fig.1 shows the relation between ν_u and K_f. It is found that assumption of undrain condition is equivalent to approach the value of Poisson's ratio to 0.5.

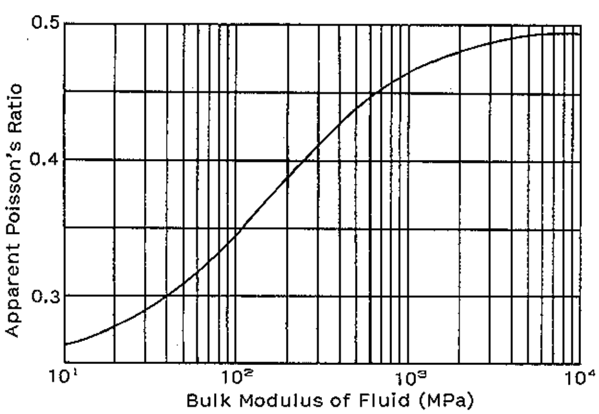

Fig.1 Relation between the bulk modulus of fluid material and apparent Poisson's ratio under undrain condition.
($n=0.375$, $K_s=37000$MPa, $\nu=0.25$, $G=81.63$MPa)

APPLICATION OF THE THIN LAYER ELEMENT METHOD

The problem solved here is in x-z two dimensional linear steady state condition. Rayleigh wave motion propagating in saturated two-phase semi- infinite media is considered. For simplification the geometry is idealized by a single beam jointed to the semi-infinite layered region as shown in Fig.2. Then unit horizontal harmonic excitation is applied to the top of beam.

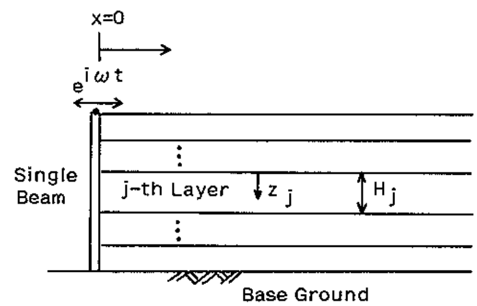

Fig.2 The system used in numerical study

Semi-infinite layered region

When the system is subjected to harmonic loading, the displacement across the layer satisfying wave equation is expressed as the following form.

$$\begin{Bmatrix}u(x,z,t)\\ w(x,z,t)\end{Bmatrix}=\exp(i\omega t)[H]\begin{Bmatrix}u(z)\\ w(z)\end{Bmatrix} \quad (13)$$

where [H] is function of x. In the case of Cartesian co-ordinates, [H] is given by (14)

$$[H]=\exp(-ihx)[I] \quad (14)$$

where [I] is a identity matrix, and h is wave number. Assuming the displacement in the j-th layer varies lineally, we can write the displacement in the j-th layer using a shape function [N]

$$\begin{Bmatrix}u(x,z,t)\\ w(x,z,t)\end{Bmatrix}_j=\exp(i\omega t)[H][N]\{u\}_{j,j+1} \quad (15)$$

in which $[N]=[(1-\eta)I,\ \eta I]$ with $0\leqq\eta=z_j/H_j\leqq 1$, and $\{u\}_{j,j+1}=\{u_j,w_j,u_{j+1},w_{j+1}\}$ is the displacement at j-th and (j+1)th interface.

Accordingly, equation(1) in j-th layer becomes

$$[W]=([K]-i\omega[C]+\omega^2[M])[H][N]\{u\}_{j,j+1}=0 \quad (16)$$

According to Galerkin's method, discretized displacement satisfies the following equation.

$$\sum_{j=1}^{M}\left[\iint_V N^T H^T[W]dV-\oint_S N^T H^T\{\sigma_b\}ds\right]=0 \quad (17)$$

in which M is a total number of layers, $\{\sigma_b\}^T = [\tau_{xz}, \sigma_z, 0, \pi]$ is the stress at the boundary s.

Integrating over the region in equation (17), the result is an eigenvalue problem of the following form

$$(h^2[A]+ih[B]+[E])\{\phi\}=0 \qquad (18)$$

Considering the equivalent nodal force at the x=0 plane, we can obtain stiffness matrix [R] across the layer in x=0 plane

$$[R]=(i[A][\phi][h]+[F][\phi])[\phi_m]^{-1} \qquad (19)$$

in which [A],[B],[E] and [F] are matrix which depend on the geometry and the material properties of discrete layers, $[\phi]$= modal matrix given by solving equation (18); [h]= diagonal matrix consist of wave number that is eigenvalue of equation (18). The detail of these matrix can be found in (Kazama & Nogami, 1991).

Here, it is neccesary to note that matrix $[\phi]$ is not a square matrix, because the number of effective eigenvalues obtained from equation (18) is not the same as the degree of freedom. It relates to absence of the shear wave propagating in the fluid material.

Superposition of the stiffness matrix

The relation between the nodal force and displacement of the semi-infinite region in x=0 plane is given by

$$\begin{Bmatrix} P_{xg} \\ P_{zg} \\ P_\pi \end{Bmatrix} = \begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix} \begin{Bmatrix} u_x \\ u_z \\ w_x \end{Bmatrix} \qquad (20)$$

Introducing the condition $w_x=0$ and $P_{zg}=0$ in x=0 plane, we can write reduced stiffness matrix of the semi-infinite region

$$\{P_{xg}\}=[R_{11}-R_{12}R_{22}^{-1}R_{21}]\{u_x\} \qquad (21)$$
$$=[R_g]\{u_x\}$$

On the other hand the dynamic stiffness matrix of the beam is expressed as

$$\begin{Bmatrix} P_x \\ M \end{Bmatrix} = \begin{bmatrix} R_{xx} & R_{x\theta} \\ R_{\theta x} & R_{\theta\theta} \end{bmatrix} \begin{Bmatrix} u_x \\ \theta \end{Bmatrix} \qquad (22)$$

in which we use a consistent mass matrix for beam. Therefore, relation between the nodal force and displacement of total system becomes

$$\begin{Bmatrix} P_x+P_{xg} \\ M \end{Bmatrix} = \begin{bmatrix} R_{xx}+R_g & R_{x\theta} \\ R_{\theta x} & R_{\theta\theta} \end{bmatrix} \begin{Bmatrix} u_x \\ \theta \end{Bmatrix} \qquad (23)$$

NUMERICAL STUDY FOR THE DYNAMIC STIFFNESS

The semi-infinite ground is divided into ten layers as shown in Fig.3. Using the developed formulation, we computed steady state harmonic displacement response at the location of the excitation force. Then, the stiffness was computed as force/displacement and normalized by the static stiffness for dry soil.

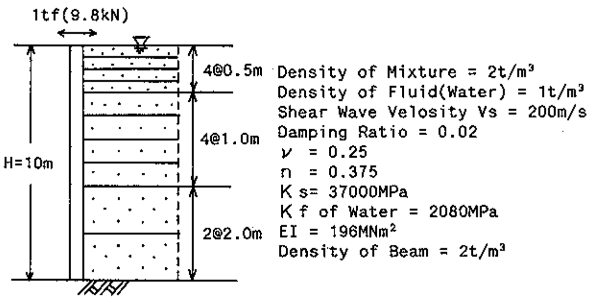

Fig.3 The geometry and physical properties

Fig.4 shows the effects of the bulk modulus of fluid material on the dynamic stiffness under undrained condition. The stiffness for saturated soil is larger than the one for dry soil since the rigidity of fluid is much larger than the one of soil skeleton. This results in increasing the resonant frequency associated with the P-wave but not affecting the resonant frequency associated with the S-wave.

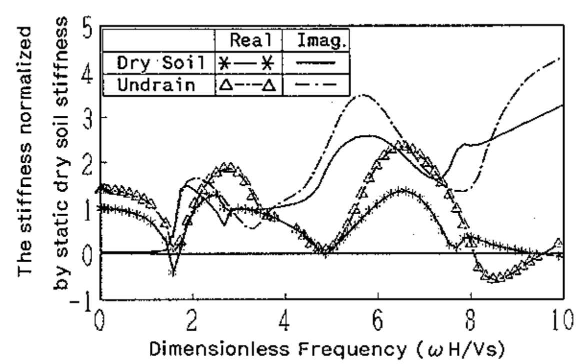

Fig.4 Comparison of the dynamic stiffness under undrained condition with one for dry soil.

Fig.5 shows the dynamic stiffness under various soil permeabilities. The coefficient of permeabilities used here are rarying from 10^{-1}m/s to 10^{-5}m/s, which correspond to from coarse gravel to fine sand. When the drainage is permitted, the pore water diffuse more easily for slower loading rate (lower frequencies) and the difference in between the dry and drained soil response diminishes completely at the static condition (frequency=0). However, the diffusion becomes more difficult as the frequency increases and the soil stiffness approaches to the one for the undrain condition. In low frequency range the maximum value of imaginary part stiffness for the drain condition is three times as large as the one for dry soil. It indicates that the system under drained condition has a large damping factor apparently. This seems to be influenced by the relative motion of pore water to soil skeleton.

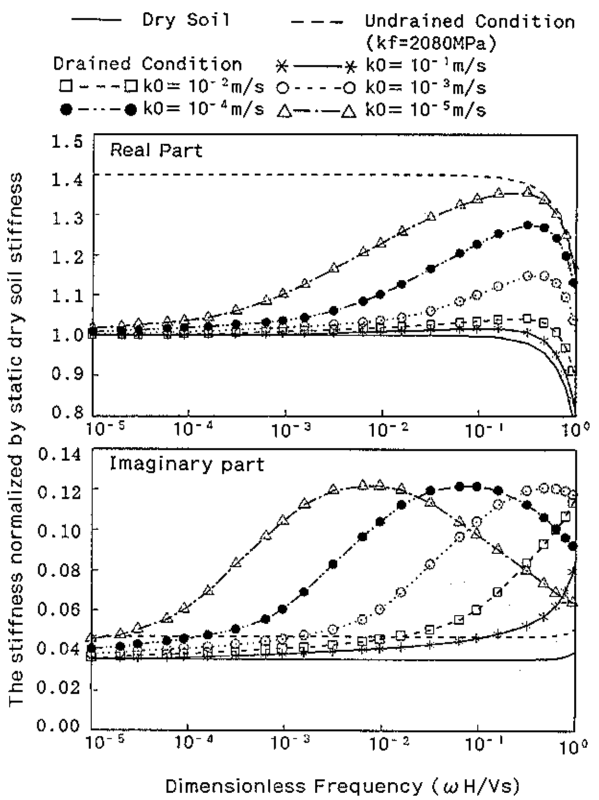

Fig.5 Effects of the permeability on the dynamic stiffness.

Distribution of the stress amplitude

Fig.6 shows the distribution of the stress amplitude in x=0 plane under various permeabilities. The stress is calculated at the middle depth of the each layer. It is found that the pore water pressure increases as the permeability decreases. Especially at the position of the near surface, in spite of large value of the total stress amplitude, the pore water pressure amplitude is not so large in the case of permeability less than 10^{-3}m/s. This is also due to rapid diffusion rate.

In this analysis the volume change due to pure shear stress is not taken into consideration. The reason is that there is no volumetric strain due to pure shear stress under elastic condition. Thus, the principle of the pore water pressure generation is different from that caused by one-dimensional shear wave motion. Under Rayleigh wave stress condition the pore water pressure varies in accordance with variation of the total stress. During actual earthquake complex stresses caused by both body wave and surface wave motion generates the pore water pressure.

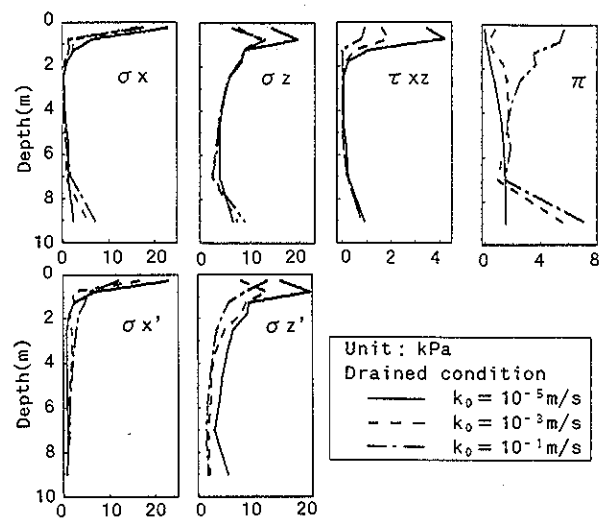

Fig.6 Effects of the permeability on the distribution of the stress amplitude with depth at dimensionless frequency(ω H/Vs)=2.

CONCLUSION

The thin layer element describing the dynamic behavior for saturated two-phase layered media was developed. Using the developed formulation, we studied the dynamic response of saturated soil comparing with that of dry soil. It was found that the permeability and loading rate (loading frequency) are one of the critical factor to control the dynamic behavior of saturated soil. It was also found that the pore water pressure caused by Rayleigh wave motion is significantly influenced by the permeability of the ground. However, the stress condition caused by Rayleigh wave motion is different from that caused by one-dimensional shear wave motion.

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